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One-Loop NMHV Amplitudes involving Gluinos and Scalars in $\mathcal{N}$=4 Gauge Theory

Steven J. Bidder[1], **Warren B. Perkins**[1] and **Kasper Risager**[2]

[1] *Department of Physics, University of Wales Swansea, Swansea, SA2 8PP, UK*
[2] *Niels Bohr Institute, University of Copenhagen, Blegdamsvej 17, DK–2100 Copenhagen ø, Denmark*

Abstract: We use Supersymmetric Ward Identities and quadruple cuts to generate n-pt NMHV amplitudes involving gluinos and adjoint scalars from purely gluonic amplitudes. We present a set of factors that can be used to generate one-loop NMHV amplitudes involving gluinos or adjoint scalars in $\mathcal{N} = 4$ Super Yang-Mills from the corresponding purely gluonic amplitude.

Keywords: Extended Supersymmetry, NLO computations.

Contents

1. Introduction

The discovery of a possible duality between gauge theory and twistor string theory [1, 2], has led to considerable progress in obtaining gauge theory amplitudes in compact forms [3, 4, 5, 6, 7, 8, 9, 10]. While most of the applications to loop calculations have been to amplitudes which involve only external gluons, 6-pt amplitudes involving adjoint fermions and scalars have been generated using Supersymmetric Ward Identities (SWI) [11] and superspace constructions [12].

In this paper we use SWI [13] and generalised unitarity cuts [5] to find one-loop n-pt "Next-to-Maximally-Helicity-Violating" or NMHV amplitudes involving adjoint fermions and scalars in $\mathcal{N} = 4$ gauge theory. The amplitudes with purely gluonic external legs were calculated in [4] and we present our results as a set of *conversion factors* that relate these purely gluonic amplitudes to those with external fermions/scalars.

In section 2 we review the structure of $\mathcal{N} = 4$ amplitudes and the recent developments in calculating them in compact forms. The purely gluonic amplitudes of [4] are reviewed in section 3 and the conversion factors calculated in section 4. In section 5 we describe how to compound these factors to find amplitudes with multiple external fermions and scalars.

2. $\mathcal{N}=4$ Amplitudes

Tree-level amplitudes for $U(N_c)$ or $SU(N_c)$ gauge theories with n external adjoint particles can be decomposed into colour-ordered partial amplitudes multiplied by an associated colour-trace [14, 15]. Summing over all non-cyclic permutations reconstructs the full amplitude $\mathcal{A}_n^{\rm tree}$ from the partial amplitudes $A_n^{\rm tree}(\sigma)$,

$$\mathcal{A}_n^{\rm tree}(\{k_i,a_i\}) = g^{n-2} \sum_{\sigma\in S_n/Z_n} {\rm Tr}(T^{a_{\sigma(1)}}\cdots T^{a_{\sigma(n)}})\ A_n^{\rm tree}(k_{\sigma(1)},\ldots,k_{\sigma(n)}) \ , \tag{2.1}$$

where k_i and a_i are respectively the momentum and colour-index of the i-th external particle, g is the coupling constant and S_n/Z_n is the set of non-cyclic permutations of $\{1,\ldots,n\}$. The $U(N_c)$ ($SU(N_c)$) generators T^a are the set of traceless hermitian $N_c\times N_c$ matrices, normalised such that ${\rm Tr}\left(T^aT^b\right)=\delta^{ab}$. Conventionally we take all particles to be outgoing. We denote gluons by g_i and adjoint fermions by Λ_i. We will often refer to the adjoint fermions as gluinos for simplicity.

The simplest non-vanishing amplitudes are the 'maximally helicity violating' (MHV) amplitudes with two particles of negative helicity and the remainder positive. The MHV partial amplitudes for gluons are given by the Parke-Taylor formulae [16],

$$A_n^{\rm tree}(g_1^+,\ldots,g_j^-,\ldots,g_k^-,\ldots,g_n^+) =\ i\,\frac{\langle j\,k\rangle^4}{\langle 1\,2\rangle\,\langle 2\,3\rangle\cdots\langle n\,1\rangle} \ , \tag{2.2}$$

for a partial amplitude where j and k are the legs with negative helicity. We use the notation $\langle j\,l\rangle\equiv\langle j^-|l^+\rangle$, $[j\,l]\equiv\langle j^+|l^-\rangle$, with $|i^\pm\rangle$ being a massless Weyl spinor with momentum k_i and chirality $\pm$ [17, 15]. The spinor products are related to momentum invariants by $\langle i\,j\rangle\,[j\,i]=2k_i\cdot k_j\equiv s_{ij}$ with $\langle i\,j\rangle^*=[j\,i]$.

The colour decomposition for one-loop amplitudes of adjoint particles takes the form [18],

$$\mathcal{A}_n^{1-\rm loop}\left(\{k_i,a_i\}\right) = g^n \sum_{c=1}^{\lfloor n/2\rfloor+1}\ \sum_{\sigma\in S_n/S_{n;c}} {\rm Gr}_{n;c}\left(\sigma\right)\ A_{n;c}(\sigma), \tag{2.3}$$

where $\lfloor x\rfloor$ is the largest integer less than or equal to x. The leading colour-structure factor,

$${\rm Gr}_{n;1}(1) = N_c\ \ {\rm Tr}\left(T^{a_1}\cdots T^{a_n}\right) \ , \tag{2.4}$$

is just N_c times the tree colour factor, and the subleading colour structures ($c>1$) are given by,

$${\rm Gr}_{n;c}(1) =\ {\rm Tr}\left(T^{a_1}\cdots T^{a_{c-1}}\right)\ {\rm Tr}\left(T^{a_c}\cdots T^{a_n}\right) \ . \tag{2.5}$$

S_n is the set of all permutations of n objects and $S_{n;c}$ is the subset leaving ${\rm Gr}_{n;c}$ invariant. Once again it is convenient to use $U(N_c)$ matrices; the extra $U(1)$ decouples [18].

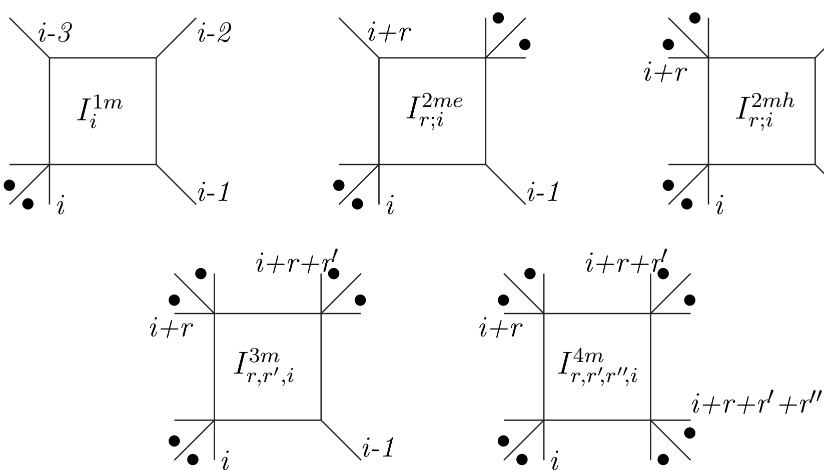
For one-loop amplitudes the subleading in colour amplitudes $A_{n;c}$, $c>1$, may be obtained from summations of permutations of the leading in colour amplitude [19].

Hence, we need only focus on the leading in colour amplitude $A_{n;1}$, which we will generally abbreviate to A_n.

One-loop amplitudes depend on the particles circulating in the loop. In $\mathcal{N}=4$ SYM cancellations between fermionic loops and bosonic loops lead to considerable simplifications in the loop momentum integrals. This is manifest in the "string-based approach" to computing loop amplitudes [20]. As a result, $\mathcal{N}=4$ one-loop amplitudes can be expressed simply as a sum of scalar box-integral functions [19],

$$I_i^{1m} \quad I_{r;i}^{2me} \quad I_{r;i}^{2mh} \quad I_{r,r',i}^{3m} \quad I_{r,r',r'',i}^{4m} \tag{2.6}$$

with the labeling as indicated,



Explicit forms for these scalar box integrals can be found in [19]. It is convenient to define dimension zero F-functions, F_i, by removing the momentum prefactors of these scalar boxes [21]. The one-loop amplitudes can then be expressed as,

$$A^{\mathcal{N}=4} = \sum_i c_i F_i \,, \tag{2.7}$$

and the computation of one-loop $\mathcal{N}=4$ amplitudes is just a matter of determining the coefficients c_i. These remarkable simplifications also appear to extend beyond one-loop [22].

These $\mathcal{N}=4$ amplitudes are related to purely gluonic amplitudes via a sum over contributions from various matter supermultiplets:

$$A_n \;\equiv\; A_n^{\mathcal{N}=4} - 4A_n^{\mathcal{N}=1 \text{ chiral}} + A_n^{[0]} \,, \tag{2.8}$$

where $A_n^{[0]}$ is the contribution from the complex scalar (or $\mathcal{N}=0$ matter multiplet) circulating in the loop. (Throughout we assume the use of a supersymmetry preserving regulator [23, 20, 24].)

Progress in calculating amplitudes has been remarkable and varied. At tree level, inspired by the duality between topological string theory and gauge theory [1], a reformulation of perturbation theory in terms of MHV-vertices was proposed [2]. This promoted the MHV amplitudes of eq. (2.2) to the role of fundamental building blocks in the perturbative expansion. By continuing legs off-shell in a well specified manner these can be sewn together to form other amplitudes. This reformulation produces relatively compact expressions for tree amplitudes. While initially presented for purely gluonic amplitudes, it has been successfully extended to other particle types [25, 26].

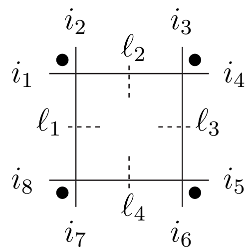
In a different development, a series of recursion relations for calculating tree amplitudes have been introduced [27]. These yield compact expressions for gluonic tree amplitudes [28], the six-point NMHV amplitudes involving fermions [29] and gravity amplitudes [30, 31, 32].

For one-loop amplitudes in gauge theory, full results for all helicities and all particle types are only known for the four-point [33, 24] and five-point [34, 35, 36, 37] amplitudes. Recently recursive techniques have been used to obtain certain 6-pt and 7-pt MHV one-loop amplitudes in QCD [38]. Beyond five-point, the one-loop amplitudes are much better understood within supersymmetric theories. Here the amplitudes are "cut-constructible", in that the coefficients can be determined from unitarity cuts. Using this fact, in [19] the one-loop amplitudes were determined for the all-n MHV amplitudes and in [21] the remaining six-point gluonic amplitudes (the NMHV amplitudes) were computed and the MHV amplitudes determined in $\mathcal{N}=1$ theories. Recursive techniques have been used to generate the n-pt one-loop split-helicity amplitudes [39].

The MHV vertex approach has also been shown to extend to one-loop in [40], where the one-loop $\mathcal{N}=4$ MHV amplitudes were computed and shown to be in complete agreement with the results of [19], and in [41, 42] where the $\mathcal{N}=1$ MHV one-loop amplitudes were computed and shown to be in agreement with the results of [21].

These techniques have been very successful and results include the recent computation of all $\mathcal{N}=4$ NMHV one-loop amplitudes with external glue [3, 4] and various next-to-next-to-MHV (N^2MHV) box-coefficients [5].

An important development which enhances the power of the unitarity method, is the observation by Britto, Cachazo and Feng [5] that box integral coefficients can be obtained from generalised unitarity cuts [43, 44, 3] by analytically continuing the massless corners of the quadruple cuts. The quadruple cuts give each box-coefficient as a product of four tree amplitudes. Applying this to the box,



where dashed lines represent cuts and dots represent arbitrary numbers of external line insertions, the box-coefficient is given by,

$$c = \frac{1}{2}\sum_{\mathcal{S}}\Big(A^{\text{tree}}(\ell_1, i_1, \ldots, i_2, \ell_2) \times A^{\text{tree}}(\ell_2, i_3, \ldots, i_4, \ell_3) \\ \times A^{\text{tree}}(\ell_3, i_5, \ldots, i_6, \ell_4) \times A^{\text{tree}}(\ell_4, i_7, \ldots, i_8, \ell_1)\Big)\ , \tag{2.9}$$

where the sum is over all allowed intermediate configurations and particle types [5] and the cut legs are frozen in a specific manner.

These techniques are also useful in calculating amplitudes in $\mathcal{N} < 4$ theories [8, 45, 9, 10], although these amplitudes are more complicated and contain integral functions other than the box functions. Unfortunately, non-supersymmetric theories are not cut-constructible [21], so the unitary techniques are not immediately applicable, although progress is ongoing in this area [46, 47].

Amplitudes with external glue can be related to ones with external fermions and scalars using Supersymmetric Ward Identities (SWI) [11]. The SWI can be obtained by acting with the supersymmetry generator $Q(\eta)$ (where η is an arbitrary spinor parameter) on a string of operators, z_i, which has vanishing vacuum expectation value. Since $Q(\eta)$ annihilates the vacuum we obtain,

$$0 = \Big\langle \Big[Q(\eta), \prod_i z_i\Big]\Big\rangle = \sum_i \Big\langle z_1 \cdots [Q(\eta), z_i] \cdots z_n \Big\rangle\,. \tag{2.10}$$

For $\mathcal{N} = 1$ supersymmetry we can use the supersymmetry algebra,

$$\begin{aligned} [Q(\eta), g^+(p)] &= -\,[\eta\, p]\,\bar{\Lambda}^+, \quad [Q(\eta), g^-(p)] = \langle p\, \eta\rangle\, \Lambda^-, \\ [Q(\eta), \bar{\Lambda}^+(p)] &= -\,\langle p\, \eta\rangle\, g^+, \quad [Q(\eta), \Lambda^-(p)] = [\eta\, p]\, g^-, \end{aligned} \tag{2.11}$$

where $g^{\pm}(p)$ is the operator creating a gluon of momentum p and $\Lambda^-(p)$ that for a gluino. Applying this to $A'_n(g_1^-, g_2^-, \Lambda_3^+, g_4^+, \ldots, g_n^+)$ we obtain,

$$0 = \langle 1\, \eta\rangle\, A_n(\Lambda_1^-, g_2^-, \bar{\Lambda}_3^+, g_4^+, \ldots, g_n^+) + \langle 2\, \eta\rangle\, A_n(g_1^-, \Lambda_2^-, \bar{\Lambda}_3^+, g_4^+, \ldots, g_n^+) \\ -\, \langle 3\, \eta\rangle\, A_n(g_1^-, g_2^-, g_3^+, g_4^+, \ldots, g_n^+)\,, \tag{2.12}$$

where we have used the fact that amplitudes with two fermions of the same helicity vanish. Appropriate choices of η then give the MHV two-gluino amplitudes directly in terms of the purely gluonic amplitude.

For NMHV amplitudes there are typically four amplitudes contributing to each identity. The system has rank 2, so it can only directly give two of the amplitudes in terms of the other two. In [11] symmetry arguments were used to resolve these ambiguities and solve the SWI at 6-pt. Alternatively, a second amplitude can be calculated explicitly and the SWI used to generate the other two. This is the approach we adopt in this paper. SWI apply order by order in perturbation theory and, as the box-integral functions are a set of independent functions, box by box.

Recently very elegant expressions for the 6-pt amplitudes have been derived using a superspace construction [12].

3. Summary of NMHV Gluonic Amplitudes

In this section we review the n-pt one-loop NMHV gluonic amplitudes derived in [4]. Our gluino amplitudes will be derived from these.

In any one-loop NMHV box diagram there are seven legs with negative helicity: three external and four internal. As each massive corner of the box requires at least two legs with negative helicity to be non-zero, we can have at most three massive corners. Further, the three-mass boxes have a particularly simple form, with three massive MHV corners and one massless $\overline{\text{MHV}}$ (or googly) corner. Thus they are 'MHV-deconstructible' in that they can be determined using purely MHV vertices and, in this case, quadruple cuts. The three-mass box-coefficient $c^{3m}(m_1, m_2, m_3; A, B, C, d)$ where A, B and C are the massive corners, d is the massless corner and m_1, m_2 and m_3 are the legs with negative helicity, is given by [4],

$$\begin{aligned} &c^{3m}(m_1, m_2, m_3; A, B, C, d) = \\ &\quad \frac{[\mathcal{H}(m_1,m_2,m_3;A,B,C,d)]^4}{\langle 1\,2\rangle\langle 2\,3\rangle\ldots\langle n\,1\rangle K_B^2} \frac{\langle A_{-1}\,B_1\rangle}{\langle d^-|\not{K}_C\not{K}_B|A_{-1}^+\rangle\langle d^-|\not{K}_C\not{K}_B|B_1^+\rangle} \frac{\langle B_{-1}\,C_1\rangle}{\langle d^-|\not{K}_A\not{K}_B|B_{-1}^+\rangle\langle d^-|\not{K}_A\not{K}_B|C_1^+\rangle}, \end{aligned} \tag{3.1}$$

where A_1 denotes the first leg of corner A and A_{-1} the last. When leg d has positive helicity, $\mathcal{H}$ is given by,

$$\begin{aligned} \mathcal{H} &= 0, && m_{1,2,3} \in A, \\ &= 0, && m_{1,2,3} \in B, \\ &= \langle m_1\,m_2\rangle\,\langle d^-|\not{K}_C\not{K}_B|m_3^+\rangle, && m_{1,2} \in A,\ m_3 \in B, \\ &= \langle m_2\,m_3\rangle\,\langle d^-|\not{K}_C\not{K}_B|m_1^+\rangle, && m_1 \in A,\ m_{2,3} \in B, \\ &= \langle m_1\,m_2\rangle\,\langle d\,m_3\rangle\,K_B^2, && m_{1,2} \in A,\ m_3 \in C, \\ &= \langle m_1\,m_2\rangle\,\langle d^-|\not{K}_A\not{K}_B|m_3^+\rangle \\ &\quad + \langle m_3\,m_2\rangle\,\langle d^-|\not{K}_C\not{K}_B|m_1^+\rangle, && m_1 \in A,\ m_2 \in B,\ m_3 \in C, \end{aligned} \tag{3.2}$$

and when leg d has negative helicity, $d = m_3$, $\mathcal{H}$ is given by,

$$\begin{aligned} \mathcal{H} &= 0, && m_{1,2} \in A, \\ &= \langle m_1\,m_2\rangle\,\langle d^-|\not{K}_C\not{K}_B|d^+\rangle, && m_{1,2} \in B, \\ &= \langle d\,m_1\rangle\,\langle d^-|\not{K}_C\not{K}_B|m_2^+\rangle, && m_1 \in A,\ m_2 \in B, \\ &= \langle d\,m_1\rangle\,\langle d\,m_2\rangle\,K_B^2, && m_1 \in A,\ m_2 \in C. \end{aligned} \tag{3.3}$$

The two-mass hard boxes are also MHV-deconstructible. As boxes with adjacent massless corners of the same type vanish, each non-vanishing two-mass easy box has a massless MHV corner, a massless googly corner and two massive MHV corners. Unfortunately, two-mass easy and one-mass boxes are not all MHV-deconstructible. However, all three types of box can be generated from the three-mass boxes using IR consistency arguments [4]. For the two-mass hard boxes the result is,

$$c^{2mh}(A,B,c,d) = c^{3m}(A,B,\{c\},d) + c^{3m}(A,B,c,\{d\}), \tag{3.4}$$

where lower case letters denote massless corners and $\{\}$ indicates that the corner should be thought of as the massless limit of a massive corner. This relationship has a simple interpretation in terms of the box diagrams: for each internal helicity configuration, one of the massless corners of the two-mass hard box will be MHV and can be thought of as the massless limit of a massive MHV corner. Summing over internal helicity configurations in general gives two terms. If one of the helicity configurations gives a vanishing contribution, the corresponding three-mass box-coefficient will also vanish.

The two-mass easy boxes can also be expressed in terms of three-mass boxes [4]:

$$c^{2me}(A,b,C,d) = \sum_{\hat{X},\hat{Y},\hat{Z}} c^{3m}(b,\hat{X}(d),\hat{Y},\hat{Z}) + \sum_{\hat{X},\hat{Y},\hat{Z}} c^{3m}(d,\hat{X}(b),\hat{Y},\hat{Z}), \tag{3.5}$$

where the sum is over all clusters (maintaining cyclic ordering) where $\hat{X}(a)$ contains leg a and $\hat{Y}$ is massive. Finally, the one-mass boxes are given by,

$$c^{1m}(A,b,c,d) = c^{2me}(A,b,\{c\},d) + c^{3m}(A,\{b\},c,\{d\}). \tag{3.6}$$

These relationships are based on the IR properties of the box integral functions and thus carry over directly to amplitudes involving gluinos and scalars.

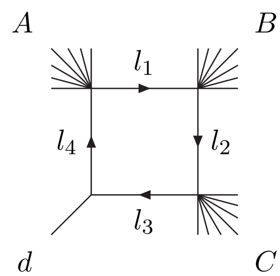
4. Conversion Factors From Supersymmetric Ward Identities and Quadruple Cuts

We first consider amplitudes with a pair of external gluinos. These are related to purely gluonic amplitudes through the SWI obtained by acting with supersymmetry generator Q on $A(g^-_{m_1}, g^-_{m_2}, g^-_{m_3}, \bar{\Lambda}^+_q, \ldots)$, where $\ldots$ represents a string of positive helicity gluons. The structure of the SWI is independent of the ordering of the legs, but there are different SWI for each distinct ordering. We will explicitly show the case where the first three legs have negative helicity. As the SWI apply box by box, we have,

$$\begin{aligned} \langle q\eta\rangle c(g^-_{m_1}, g^-_{m_2}, g^-_{m_3}, g^+_q, \ldots) = \quad & \langle m_1\eta\rangle c(\Lambda^-_{m_1}, g^-_{m_2}, g^-_{m_3}, \bar{\Lambda}^+_q, \ldots) \\ & + \langle m_2\eta\rangle c(g^-_{m_1}, \Lambda^-_{m_2}, g^-_{m_3}, \bar{\Lambda}^+_q, \ldots) \\ & + \langle m_3\eta\rangle c(g^-_{m_1}, g^-_{m_2}, \Lambda^-_{m_3}, \bar{\Lambda}^+_q, \ldots), \end{aligned} \tag{4.1}$$

where c is a generic box-coefficient.

The SWI (4.1) has rank two, so it determines two of the box-coefficients in terms of the other two. Our approach is to determine one of the two-gluino box-coefficients using quadruple cuts and then use the SWI to generate the other two. As in the purely gluonic case, we can express all of our box-coefficients as sums of the three-mass ones, so we only need to evaluate the latter explicitly. As the three-mass boxes are MHV-deconstructible, we can determine any tree amplitude we need using [48]. A generic box is shown in the figure:

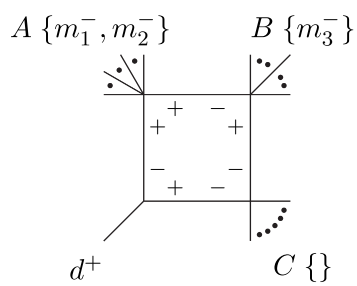


The massless corner is a 3-pt googly vertex, so we have the following useful results,

$$l_3 = l_4 + d, \qquad |l_3^+\rangle \propto |d^+\rangle, \qquad |l_4^+\rangle \propto |d^+\rangle,$$
$$\langle dl_1\rangle[l_1 l_2]\langle l_2| = \langle d^-|\not{K}_A \not{K}_B|, \qquad \langle dl_2\rangle[l_2 l_1]\langle l_1| = \langle d^-|\not{K}_C \not{K}_B|, \tag{4.2}$$

which allow us to evaluate expressions that are homogeneous in l_i.

We label the purely gluonic boxes by the location of the negative helicity legs. The 'AAB' box shown below has two negative helicity legs on corner A and one on corner B:



When we relate this purely gluonic box to one with a pair of external gluinos, we must specify which g^+ to replace by $\bar{\Lambda}^+$ and which g^- to replace by Λ^-. Using m to label the external Λ^- leg, q to label the external $\bar{\Lambda}^+$ leg and $L(q)$ to denote its location, the conversion factor is given by $R^{\text{box label}}_{L(q)m}$, so that,

$$c^{xxx}(g^-, g^-, \Lambda^-_m, \bar{\Lambda}^+_q, \ldots) = R^{xxx}_{L(q)m} c^{xxx}(g^-, g^-, g^-_m, g^+_q, \ldots). \tag{4.3}$$

The AAB box shown is an example of a 'singlet' box, where only gluons can circulate in the loop and there is a single contribution to the purely gluonic box-coefficient. We can immediately see that there is no possible routing for a fermion,

Λ^-, from corner B to corner A or d, so we have,

$$R^{AAB}_{Am_3} = R^{AAB}_{dm_3} = 0. \tag{4.4}$$

The remaining $R^{AAB}_{Am_i}$ and $R^{AAB}_{dm_i}$ then follow from the SWI. For $R^{AAB}_{Bm_i}$ and $R^{AAB}_{Cm_i}$ there are either one or two possible fermion routings and one box in each class must be calculated using quadruple cuts before the other two can be read off from the SWI. The conversion factors for the other singlet boxes can be similarly evaluated. The results of these calculations are presented in table 1.

The ABC boxes are 'non-singlet' and any particle in the $\mathcal{N} = 4$ multiplet can circulate in the loop. The purely gluonic box-coefficients are obtained by summing over diagrams with all possible particles circulating in the loop. If the $\bar{\Lambda}^+$ and Λ^- attach to the same corner, any particle can still circulate in the loop. Each corner remains MHV, but care must be taken with the flavour structure of corners with four non-gluonic legs as these amplitudes are flavour dependent. In all cases the MHV tree amplitudes can be found using [48].

Our results are presented in table 1. For each type of box the conversion factors have a common denominator. The factor appearing in each denominator also appears in the numerator of the corresponding purely gluonic amplitude, where it is raised to the fourth power. Conversion factors are presented for all distinct cases. The factors not explicitly listed can be obtained by flipping (e.g. AAB boxes flip into BCC boxes). The denominator of each conversion factor is given next to the box name and the numerators are listed for each location of q and for each m.

AAB		$\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_3^+\rangle\langle m_1 m_2\rangle$
A	m_1	$\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_3^+\rangle\langle q m_2\rangle$
$/d$	m_2	$\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_3^+\rangle\langle m_1 q\rangle$
	m_3	0
B	m_1	$-\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_2^+\rangle\langle m_3 q\rangle$
	m_2	$-\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_1^+\rangle\langle q m_3\rangle$
	m_3	$\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert q^+\rangle\langle m_1 m_2\rangle$
C	m_1	$\langle d^-\vert \slashed{K}_A \slashed{K}_B \vert q^+\rangle\langle m_2 m_3\rangle - \langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_2^+\rangle\langle m_3 q\rangle$
	m_2	$\langle d^-\vert \slashed{K}_A \slashed{K}_B \vert q^+\rangle\langle m_3 m_1\rangle - \langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_1^+\rangle\langle q m_3\rangle$
	m_3	$\langle d^-\vert \slashed{K}_A \slashed{K}_B \vert q^+\rangle\langle m_1 m_2\rangle + \langle d^-\vert \slashed{K}_C \slashed{K}_B \vert q^+\rangle\langle m_1 m_2\rangle$
		$= -K_B^2\langle dq\rangle\langle m_1 m_2\rangle$

AAC		$K_B^2\langle d m_3\rangle\langle m_1 m_2\rangle$
A	m_1	$K_B^2\langle d m_3\rangle\langle q m_2\rangle$
$/d$	m_2	$K_B^2\langle d m_3\rangle\langle m_1 q\rangle$
	m_3	0
B	m_1	$-\langle d^-\vert \slashed{K}_A \slashed{K}_B \vert m_3^+\rangle\langle q m_2\rangle + \langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_2^+\rangle\langle m_3 q\rangle$
	m_2	$-\langle d^-\vert \slashed{K}_A \slashed{K}_B \vert m_3^+\rangle\langle m_1 q\rangle + \langle d^-\vert \slashed{K}_C \slashed{K}_B \vert m_1^+\rangle\langle q m_3\rangle$
	m_3	$-\langle d^-\vert \slashed{K}_C \slashed{K}_B \vert q^+\rangle\langle m_1 m_2\rangle$
C	m_1	$-K_B^2\langle d m_2\rangle\langle m_3 q\rangle$
	m_2	$-K_B^2\langle d m_1\rangle\langle q m_3\rangle$
	m_3	$K_B^2\langle dq\rangle\langle m_1 m_2\rangle$

ABB		$-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2m_3\rangle$
A	m_1	$-\langle d^-\vert\not{K}_C\not{K}_B\vert q^+\rangle\langle m_2m_3\rangle$
$/d$	m_2	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_3^+\rangle\langle m_1q\rangle$
	m_3	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle qm_1\rangle$
B	m_1	0
	m_2	$-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle qm_3\rangle$
	m_3	$-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$
C	m_1	$-\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_2m_3\rangle$
	m_2	$\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_3m_1\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle qm_3\rangle$
	m_3	$\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_1m_2\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$

ABd		$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle dm_1\rangle$
A	m_1	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle dq\rangle$
	m_2	0
	d	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle qm_1\rangle$
B	m_1	$-\langle d^-\vert\not{K}_C\not{K}_B\vert d^+\rangle\langle qm_2\rangle$
	m_2	$\langle d^-\vert\not{K}_C\not{K}_B\vert q^+\rangle\langle dm_1\rangle$
	d	$-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$
C	m_1	$\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_2d\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert d^+\rangle\langle qm_2\rangle$
	m_2	$-K_B^2\langle dq\rangle\langle dm_1\rangle$
	d	$\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_1m_2\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$

ACd		$K_B^2\langle dm_1\rangle\langle dm_2\rangle$
A	m_1	$K_B^2\langle dm_2\rangle\langle dq\rangle$
	m_2	0
	d	$K_B^2\langle dm_2\rangle\langle qm_1\rangle$
B	m_1	$\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_2d\rangle$
	m_2	$-\langle d^-\vert\not{K}_C\not{K}_B\vert q^+\rangle\langle dm_1\rangle$
	d	$-\langle d^-\vert\not{K}_A\not{K}_B\vert m_2^+\rangle\langle qm_1\rangle+\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$
C	m_1	0
	m_2	$-K_B^2\langle dm_1\rangle\langle m_2q\rangle$
	d	$-K_B^2\langle dm_1\rangle\langle qd\rangle$

BBd		$\langle d^-\vert\not{K}_A\not{K}_B\vert d^+\rangle\langle m_1m_2\rangle$
A	m_1	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle dq\rangle$
	m_2	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle qd\rangle$
	d	$-\langle d^-\vert\not{K}_C\not{K}_B\vert q^+\rangle\langle m_1m_2\rangle$
B	m_1	$-\langle d^-\vert\not{K}_C\not{K}_B\vert d^+\rangle\langle qm_2\rangle$
	m_2	$-\langle d^-\vert\not{K}_C\not{K}_B\vert d^+\rangle\langle m_1q\rangle$
	d	0
C	m_1	$+\langle d^-\vert\not{K}_A\not{K}_B\vert m_2^+\rangle\langle dq\rangle$
	m_2	$-\langle d^-\vert\not{K}_A\not{K}_B\vert m_1^+\rangle\langle qd\rangle$
	d	$-\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_1m_2\rangle$

ABC		$\langle d^-\vert\not{K}_A\not{K}_B\vert m_3^+\rangle\langle m_1m_2\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2m_3\rangle$
A	m_1	$\langle d^-\vert\not{K}_A\not{K}_B\vert m_3^+\rangle\langle qm_2\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert q^+\rangle\langle m_2m_3\rangle$
	m_2	$-K_B^2\langle dm_3\rangle\langle m_1q\rangle$
	m_3	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle qm_1\rangle$
B	m_1	$\langle d^-\vert\not{K}_A\not{K}_B\vert m_3^+\rangle\langle qm_2\rangle$
	m_2	$\langle d^-\vert\not{K}_A\not{K}_B\vert m_3^+\rangle\langle m_1q\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle qm_3\rangle$
	m_3	$-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$
C	m_1	$-\langle d^-\vert\not{K}_A\not{K}_B\vert m_2^+\rangle\langle m_3q\rangle$
	m_2	$K_B^2\langle dm_1\rangle\langle qm_3\rangle$
	m_3	$\langle d^-\vert\not{K}_A\not{K}_B\vert q^+\rangle\langle m_1m_2\rangle-\langle d^-\vert\not{K}_C\not{K}_B\vert m_1^+\rangle\langle m_2q\rangle$
d	m_1	$-\langle d^-\vert\not{K}_A\not{K}_B\vert m_2^+\rangle\langle m_3d\rangle$
	m_2	$K_B^2\langle dm_1\rangle\langle dm_3\rangle$
	m_3	$\langle d^-\vert\not{K}_C\not{K}_B\vert m_2^+\rangle\langle dm_1\rangle$

Table 1: Numerators and Denominators For Conversion Factors R_{qm}^{xxx}

The general effect of applying one of these conversion factors is to replace the $\mathcal{H}^4$ factor in the purely gluonic box-coefficient by $\mathcal{H}^3\tilde{\mathcal{H}}$, where $\tilde{\mathcal{H}}$ is the factor appearing in the 'switched' purely gluonic box-coefficient, where leg q is a negative helicity gluon

and leg m is a positive helicity gluon. This is reminiscent of the behaviour of the MHV tree amplitudes, but in this case it appears at the level of the box-coefficients.

So far we have only considered three-mass boxes. As in the purely gluonic case, the two-mass and one-mass box-coefficients for gluinos can be expressed as sums of three-mass box-coefficients. Given that the factors appearing in the SWI are simply determined by the momenta of the legs on which the supersymmetry generator acts, we see that, when expressed in terms of three-mass boxes, any SWI for (say) a two-mass easy box is just a sum of the three-mass SWI and thus trivially satisfied. We have explicitly calculated the n-pt two-mass hard box-coefficients with two gluinos using quadruple cuts and verified the consistency of the two approaches. We have also used the 6-pt NMHV tree expressions of [11] to calculate both singlet and non-singlet example 8-pt two-mass easy box-coefficients using quadruple cuts. These results are also in agreement with those obtained by summing the appropriate three-mass coefficients.

5. Beyond Two-Fermion Amplitudes

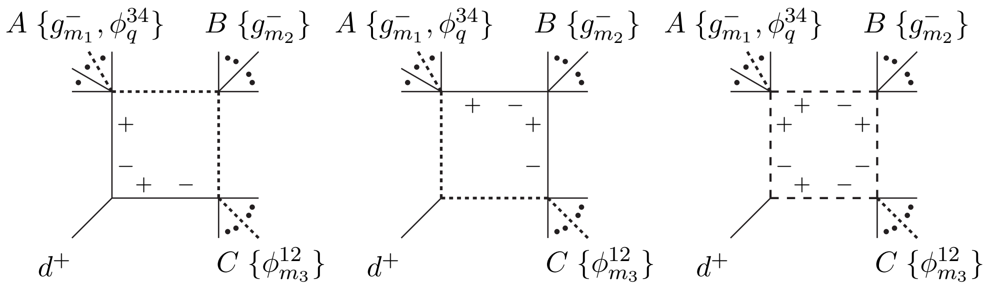
The conversion factors in table 1 can be compounded to generate amplitudes with arbitrary numbers of external adjoint scalars and fermions. The first step is to note that the box-coefficient for a diagram involving two external scalars can be obtained by simply squaring the conversion factor for the corresponding two gluino diagram,

$$c^{xxx}(g^-, g^-, \phi_m^-, \phi_q^+, \ldots) = (R^{xxx}_{L(q)m})^2 c^{xxx}(g^-, g^-, g_m^-, g_q^+, \ldots). \tag{5.1}$$

For singlet two-gluino diagrams with only one possible route for the fermion, the corresponding two-scalar diagram is obtained from the two gluino by replacing the single fermion line with a single scalar line. As we only have MHV and googly corners in the three-mass boxes, this simply gives us the square of the factor relating the two gluino box-coefficient to the gluonic. For two-gluino diagrams with two routes for the gluino, there are two-scalar diagrams where the scalar takes one of these two routes and additionally there is a diagram with a fermionic loop. The first two diagrams give factors which are the squares of the individual gluino factors, while explicit calculation shows that the last yields precisely the cross-term that arises when the sum of the gluino terms is squared. For the non-singlet diagrams, explicit calculation again shows that the two scalar box-coefficients are also simply obtained by squaring the relevant conversion factor. Recalling that we can express all of our box-coefficients in terms of the three-mass ones, we see that all the two-scalar box-coefficients are simply obtained by squaring the relevant factors in table 1.

To obtain SWI involving scalars we consider the action of a pair of supersymmetry generators Q_1 and Q_2 that generate an $\mathcal{N} = 2$ subalgebra [13, 49]. The SWI then contain amplitudes involving two flavours of gluino and a single flavour scalar. In

$\mathcal{N} = 2$ terms it is natural to denote scalar as $\phi^+ \equiv \phi_{12}$ and $\phi^- \equiv \phi_{34}$. This notation is more compact than the full $\mathcal{N} = 4$ flavour labelling, but care must be taken when counting the negative helicities required for a MHV vertex. In particular, replacing a g^+ by ϕ_{34} effectively introduces an extra negative helicity. This is important in understanding the two-scalar ABC boxes, as all three of the diagrams shown below contribute to this two-scalar box-coefficient:



In these diagrams dotted lines represent scalars, while dashed lines represent fermions.

Next we consider the NMHV amplitudes with three non-gluonic external legs. These box-coefficients are related to the purely gluonic ones by a pair of conversion factors:

$$c^{xxx}(g^-, g^-, \phi^-_m, \bar{\Lambda}^+_{q_1}, \bar{\Lambda}^+_{q_2} \ldots) = R^{xxx}_{L(q_1)m} R^{xxx}_{L(q_2)m} c^{xxx}(g^-, g^-, g^-_m, g^+_{q_1}, g^+_{q_2} \ldots), \tag{5.2}$$

$$c^{xxx}(g^-, \Lambda^-_{m_1}, \Lambda^-_{m_2}, \phi^+_q, \ldots) = R^{xxx}_{L(q)m_1} R^{xxx}_{L(q)m_2} c^{xxx}(g^-, g^-_{m_1}, g^-_{m_2}, g^+_q, \ldots). \tag{5.3}$$

For boxes with unique routings for the fermions, this result again follows directly from the form of the MHV amplitudes at each corner. For all boxes explicit calculation shows that the fermionic factors compound in this way.

Amplitudes involving four or more non-gluonic legs can now be generated directly from the appropriate SWI. We define the 'level' of an amplitude to be the number of external fermions plus twice the number of external scalars. Amplitudes with odd level will vanish and we use these as the starting points for our SWI. For example, acting with Q_2 on the level 3 amplitude, $A(\Lambda^{1-}_1, g^-_2, g^-_3, \bar{\Lambda}^{1+}_4, \bar{\Lambda}^{2+}_5, \ldots)$, gives $A(\Lambda^{1-}_1, \Lambda^{2-}_2, g^-_3, \bar{\Lambda}^{1+}_4, \bar{\Lambda}^{2+}_5, \ldots)$ and $A(\Lambda^{1-}_1, g^-_2, \Lambda^{2-}_3, \bar{\Lambda}^{1+}_4, \bar{\Lambda}^{2+}_5, \ldots)$ in terms of known amplitudes. We can now work systematically, level by level, to generate amplitudes with any number of external scalars and fermions.

6. Conclusions

One-loop NMHV amplitudes in $\mathcal{N} = 4$ gauge theory can be expressed in terms of MHV-deconstructible diagrams and so can be evaluated using quadruple cuts and known MHV tree amplitudes. These amplitudes also satisfy SWI which can be employed to minimize the number of diagrams that must be computed explicitly. We have used these techniques to determine a set of conversion factors that relate

two-gluino box-coefficients to purely gluonic ones. Analysis of quadruple cuts was then used to show how these factors can be compounded to give two-scalar and scalar-gluino-gluino box-coefficients. Amplitudes involving more external fermions/scalars then follow from SWI.

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